“Objective Priors: A Selective Review” by Malay Ghosh

Discussion by Sudip Bose
The George Washington University.

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Professor Ghosh has presented interesting features of objective priors, in particular Jeffreys' prior.

- General properties include invariance under reparameterization – he points out that invariance may also have odd or unattractive consequences; consider variance σ^2 and precision $r = (\sigma^2)^{-1}$.
- Tools – asymptotic expansion and shrinkage argument (J. K. Ghosh and colleagues)
- Divergence priors
- Matching priors

- Shrinkage argument can simplify derivation of higher order frequentist asymptotics
- Divergence priors – Bernardo/Berger reference priors are based on Kullback-Leibler divergence
- Generalization: if one considers a class of divergence measures based on β, that includes K-L divergence, Bhattacharyya-Hellinger, Chi-square, one still gets Jeffreys' prior

- Matching of Bayesian (conditioned on observed data, θ random) and frequentist (observations random, given fixed parameter θ) probabilities of intervals upto order n^{-p}; $p = 1$ first order matching, $p = 2$ second order matching
- Moment matching priors

Objective vs. Subjective priors – Bayesian Analysis (2006, Vol. 1, no. 3)

Objective:

- standard prior for a given situation
- Unification of Bayesian and frequentist
- Use both: subjective priors for some parameters, objective priors for others

Subjective:

- What the purpose of the investigation is – improving our understanding of something vs what do we learn if the data are all we have.
- Scientific learning proceeds from what we believe before an experiment to what we believe after an experiment

An argument of Walley and others against improper priors

Consider for example uniform prior on $\theta \in (-\infty, \infty)$. Let $B = (-10^{10}, 10^{10})$ be a long finite interval. With uniform prior, you are willing to bet at any odds that θ is **not** in B. (Behavioral interpretation of improper priors: Hartigan 1983)

Walley suggests lower and upper probabilities for sets, $p_1 \leq P(A) \leq p_2$.

Closely related to Bayesian robustness.

Perhaps class of priors such as $L(\theta) \leq \pi(\theta) \leq U(\theta)$ with $L(\theta)$ standard normal, $U(\theta) = 1$.

Powers of $I(\theta)$? vary criterion, vary parameterization.